SFQ Single-Shot Readout Circuit for a Flux Qubit Based on Current-to-Time Conversion

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***Abstract*—We study on a single-shot single-flux-quantum (SFQ) readout circuit based on current-to-time conversion for superconducting flux qubits. Circulating currents representing states of a qubit were converted to propagation delay time of SFQ pulses, and then detected by a time discriminator. The key parameter which determines the availability of single-shot readout is current resolution. We evaluated the current resolution by measuring the gray zone which was defined as the transition width in the output probability of the current-to-time converter with 1000 trials for each value of an input current. For reducing unwanted effects on the qubit, shunt resisters are required to be removed for Josephson junctions in the readout circuit. We designed two readout circuits for comparison, the circuit composed of shunted junctions and that of unshunted junctions. Gray zone was measured to be 4.8 μA for unshunted junctions and 2.2 μA for the shunted ones at 4.2 K. Large gray zones might originates from $1/f$ noise. Operation at a reduced temperature and employment of a likelihood decision circuit will lead to reduced gray zone, i.e., improved current resolution sufficient for single-shot readout.**

***Index Terms*—flux qubit, single-shot readout, time-to-digital conversion**

I. INTRODUCTION

In recent years, we have many reports on superconducting quantum bits (qubits) to make quantum computers. Peripheral circuits which control/observe quantum states of superconducting qubits are indispensable for building quantum computers. Superconducting fulx qubits attract the attention because of the longest coherece time among several superconducting qubits [2], [3]. In many cases reported so far, the measurement of qubit states has been performed by using a dc-SQUID. However, the dc-SQUID generates relatively large heat, so that it is difficult to integrate many dc-SQUIDs for multiple qubits. For future quantum computers based on the flux qubits, another readout circuit having both the feature of high-speed and low power dissipation is required.

A readout technique based on SFQ circuit [1] is one of the most promising candidates which satisfy the above-mentioned features. Thus, we have studied an SFQ readout circuit and proposed a new method based on the current-to-time conversion [7]. The developed readout circuit works as a current comparator which detects the direction of circulating currents representing quantum states of a flux qubit. Although several SFQ-based readout circuits have been proposed to date [4], [5], [6], only our readout circuit has a symmetric structure similar to a dc-SQUID composed of two identical junctions. Thus, sensitivity comparable to that of dc-SQUID is expected. In addition, the symmetric structure brings less effect on quantum states. In our readout circuit, the most important parameter which determines availability of single-shot readout is current resolution. We have evaluated time jitters in SFQ circuits so far [7].

In this article, we report experimental results of the current resolution for our readout circuits designed based on measured time jitters. We also discuss availability of single-shot readout.

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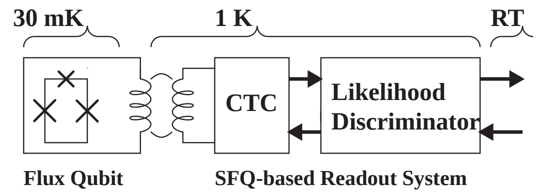


Fig. 1. Schematic diagram of implimentation as SFQ readout system we are assuming. flux qubits are placed at 30 mK stage and SFQ-base readout systems operate at 1 K stage on a refrigerator.

II. QUBIT READOUT SYSTEM

Fig. 1 is an overview of our SFQ-based readout system for flux qubits. Here, we assume that a flux qubit is placed at 30 mK and our readout system is at 1 K. This readout system has two components, a current-to-time converter (CTC) and a likelihood discriminator. The CTC outputs one or no SFQ pulse depending on the circulating current in a flux qubit. The likelihood discriminator reduces the gray zone, i.e. improves the current sensitivity by decision based on the switching probability of the CTC. As described in the introduction, the current resolution of the CTC is the most important parameter from a point of view of single-shot readout in this system.

Fig. 2 shows the schematic diagram of the CTC. The circulating current of a qubit induces a current at the splitter and split into two through a pickup coil placed between the qubit and the CTC. An SFQ for readout called 'readout SFQ' in the study is provided to the splitter and split into two different paths as shown in the figure. If a circulating current has a clockwise direction, one SFQ pulse is accelerated in the upper path 'A' of the splitter, while the other is decelerated in the lower path 'B'. The accelerated pulse reaches the 'clk' port of the DFF and the decelerated one reaches the 'din' port. A positive time difference Δt between the two SFQ pulses is generated at the DFF. For a counter-clock wise circulating current, the DFF is tuned so as to output no SFQ pulse called 'output SFQ' from the 'dout' port. In this case Δt is negative.

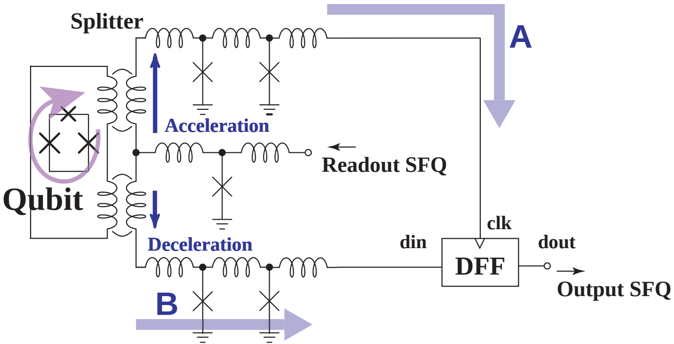


Fig. 2. Schematic diagram of SFQ readout circuit based on current-to-time conversion (Current-to-Time Converter; CTC) we have proposed. In this figure, circulating current of a qubit is CW and the CTC outputs no SFQ pulse. Crosses represent Josephson junctions.

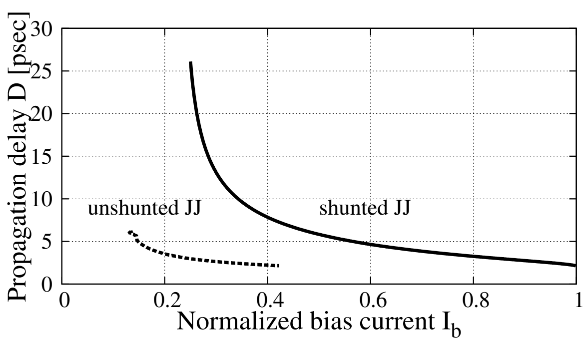


Fig. 3. Propagation delay of a SFQ pulse in a shunted and a unshunted JTL caluculated numerically. These plots show the delay time of one Josephson junction on a JTL.

The DFF discriminates Δt, namely, the DFF serves as a time-discriminator. The actual CTC has finite current resolution since noise causes time jitters in the circuit. There is so-called 'gray zone' which appears in the transition curve of the output probability of DFF for an input current flowing at the inductance coupled to the splitter. We defined the current resolution as the transition width in the curve. Probability in the gray zone is expressed using the error function,

$$P(I_{exp}) = \frac{1}{2} erf\left(\frac{I_{exp} - I_{avg}}{\sqrt{2}\sigma_{exp}}\right), \tag{1}$$

where I_{avg} is expectation of current and σ_{exp} is standard deviation obtained by fitting of experimental values, $P(I_{exp})$ and I_{exp}. The width of gray zone ΔI equal to the current resolution is defined by

$$\Delta I = \sigma_{exp}\sqrt{2\pi}. \tag{2}$$

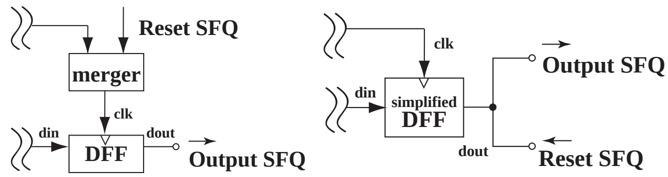
III. Circuit Design

A. Estimation of Time Jitter

Time jitters accumulate on paths from the splitter to the discriminator (balanced comparator inside the DFF). The total time jitter σ_{all} can be expressed as ,

$$\sigma_{all} = \sqrt{(n_s\sigma_s)^2 + (n_{us}\sigma_{us})^2 + (n_{cp}\sigma_{cp})^2}. \tag{3}$$

Here σ_s, σ_{us} and σ_{cp} are time jitter of a shunted JTL, an unshunted JTL and a balanced comparator, respectively. n_s, n_{us} and n_{cp} are the number of these element in the circuit. Fig. 3 shows the numerically obtained propagation delays D as a function of the normalized bias current $I_b = i_b/I_c$, where i_b is the value of bias currents and I_c is the value of critical currents of a junction on a JTL. The calculation is done for two type of junctions made up of 100 μA shunted/unshunted junctions with the parameters of the SRL standard fabrication process by JSIM [9]. Low bias currents provide large delays both for unshunted junctions and shunted junctions. The shunted junctions have much larger delay than the unshunted ones at maximum. In our CTC, the dD/dI_b is the most important factor. This derivative is also much larger in the shunted junctions than in the unshunted ones.



(a) Conventional DFF (b) Simplified DFF

Fig. 4. A conventional DFF and the simplified DFF we desined in this study. Use of conventional DFF for our CTC induce to add the merger, which has remarkable time jitter. The simplified DFF can be removed these time jitter source.

B. Splitter

The time difference depends on the bias current is provided to the splitter. When we set the operating point of the bias current to be low, the time difference generated by the splitter is increased. Increased inductance of the splitter in exchange for low I_c of the junctions leads to large induced current, resulting in larger time difference. The low I_c means to reduce Johnson noise due to increased resistor of Josephson junction [7].

C. DFF

Fig. 4(b) and Fig. 4(a) show the configurations of the DFF designed for the CTC. In general, a DFF has an internal state reflecting the storage of an SFQ. Thus it is necessary to provide an another SFQ for reset called 'reset SFQ' here to erase the stored SFQ for initialization in readout process. An additional merger is required to be placed before 'clock' port of the DFF as indicated Fig. 4(a). The merger has a balanced comparator, so that the total number of the balanced comparator becomes two. Increase in that number is critical problem for the current resolution of the CTC because the balanced comparator has very large time jitter as described later. To reduce the number, we introduced a simplified DFF shown in Fig. 4(b). For this DFF, we provided a reset SFQ after sending a readout SFQ. The reset SFQ always appears at the output port of the CTC after a given time delay from the input of a readout SFQ.

In our final design, we employ unshunted junctions except for the DFF. The values used in the (3) are $n_{us} = 10,\ n_s = 4$

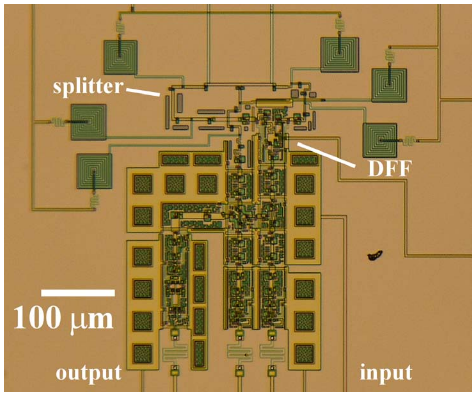


Fig. 5. Microphotograph of the actual SFQ readout circuit for testing. This circuit was fabricated with NEC Nb 2.5 kA/cm^2 process [8].

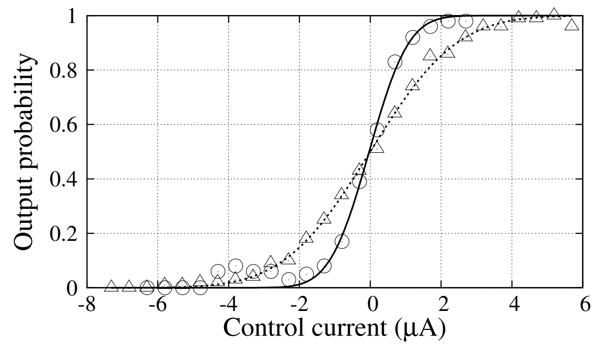


Fig. 6. Typical experimental results of gray zone measurements. The plot shows the gray zone of unshunted CTC (triangle) and the shunted CTC (circle). The dashed line and the solid line indicate function fitted by error function.

TABLE I
EXPERIMENTAL RESULTS OF TEST CIRCUITS

circuit	I_b	I_{avg}	ΔI
primitive CTC	0.75	32 μA	34.6 μA
unshunted CTC	0.35	-0.7 μA	4.8 μA
shunted CTC	0.57	-36 μA	2.2 μA
QOS comparator	-	4.15 μA	3.3 μA

and $n_{cp} = 1$. The previous experimental results show $\sigma_s = 65$ fs and $\sigma_{cp} = 365$ fs [7]. σ_{us} is estimated as 10 fs. By using those, we can estimate $\sigma_{all} = 459$ fs from (3).

Fig. 5 shows the microphotograph of the readout circuit. This SFQ readout circuit is designed using CONNECT cell library [10] basically. The splitter, simplified DFF and unshunted junctions are redesigned for this study.

IV. EXPERIMENTAL

We evaluate the current resolution of the CTC by measuring its gray zone. The measurement is performed at 4.2 K. We need to measure the output possibility, thus we send 1000 pairs of a readout SFQ and reset SFQ, and record waveforms of the input signals and of output ones for one given current flowing in the pickup coil. The current is provided externally and changed in sufficiently wide range. We call this current the control current here. To suppress external noise generated at room temperature, we place low-pass filters and copper powder filters for all the cables. The output probability can be plotted for each control current, and a transition region i.e. a gray zone appears in the plot. Fig. 6 shows experimental results for the CTC based on our final design expressed as unshunted CTC in the figure. The plots are fitted to (1), and gray zone is caluculated to be 4.8 μA. We also made two other CTCs for comparison. One is named a primitive CTC designed based on the CONNECT cell library. No techniques to improve current resolution are introduced to the primitive CTC. The other is named the shunted CTC in which shunted junctions are used in the whole circuit. In addition to those, we also tested a quasi-one-junction (QOS) comparator [11] for comparison because a QOS has a high potential in current sensitivity for a single-shot readout although its configuration is asymmetric. The gray zone of shunted CTC is also displayed in Fig. 6. The gray zone is 2.2 μA. These gray zones are normalized by each I_{avg} for clarity. Experimentally used parameters I_b, I_{avg} and obtained ΔI are summarized in TABLE I.

V. DISCUSSION

The primitive CTC showed the widest gray zone in the CTCs studied here. When we compare in gray zone between unshunted CTC and shunted CTC, the shunted CTC has narrower grayzone 2.2 μA than unshunted CTC of 4.8 μA, although time jitter σ_{all} thought to be larger than that of unshunted CTC. In our CTC, $\Delta I_b \equiv \sigma_{all}/\frac{dD}{dI_b}|_{I_b=I_{be}}$ is the most important factor, where I_{be} is the experimentally used value of I_b as indicated in Fig. 3. ΔI_b in shunted CTC is smaller than in unshunted one. As a result, shunted CTC has better current resolution. This means that low bias currents and the introduction of the simplified DFF are effective for reduction of the gray zone, i.e. improvement in current resolution.

Previously, we have considered that Johnson noise is dominant noise on the circuit. However, we have found that the gray zone of the circuit would be significantly-affected by $1/f$ noise from experimental results of the QOS [11]. Although $\Delta I = 3.3$ μA of the QOS was observed in this method, another experiment with applying AC signal as the control current showed about 1 μA of gray zone. We are not able to specify the origin of this noise yet. In an actual readout of a flux qubit, much higher repetition rate will be used for the likelihood discrimination shown in Fig. 1. Thus, current resolution is possibly improved.

Here we discuss the availability of single-shot readout in the SFQ-based readout system. In this time, the gray zone of the CTC measured at 4.2 K is insufficient for the requirement $\Delta I < 1$ μA. As described above, observable noises at 4.2 K are thought to be Johnson noise and $1/f$ noise. However, we should consider Johnson noise dominantly, because $1/f$ noise is remarkably reduced at lowered temperatures and the magnitude of $1/f$ noise becomes small in an actual readout as described above. The standard deviation of current noise (Johnson noise) is known to be proportional to the square root of temperature. Thus gray zone of the CTC is expected to be half when the readout system operates at 1 K on a refrigerator. Another important factor concerning single-shot readout is the coupling coefficient of the transformer between the input coil and the splitter. The present coefficient is measured to be 0.2. We successfully increased the coefficient into 0.8 by forming a hole in the ground plane below the transformer. Employing lower critical current junction on the splitter also improve the current resolution since low I_c of the junctions leads larger time delay of SFQ pulses. Furthermore, a numerical analysis shows that the current resolution is improved by about one order of magnitude by using the likelihood discriminator with about 250 times readouts. We can estimate that the combination of these methods make the readout system possible to operate with single-shot readout.

VI. Conclusion

We have proposed SFQ-based readout system for a flux qubit toward single-shot readout. A CTC which is the key component of this system have been designed besed on time jitters we have measured. The CTCs consist of some techniques to achieve higher current resolution; low biasing of the splitter, reducing number of comparator in simplified DFF. We have evaluated the current resolution by measuring gray zone at 4.2 K and have compared with other CTCs and a QOS comparator. Gray zone was measured to be 4.8 μA for unshunted CTC and 2.2 μA for the shunted ones. We have found that $1/f$ noise cases large gray zone from comparison with experimental result of QOS comparator. The single-shot readout of the fulx qubit can be performed when the system composing CTC and likelihood discriminator operates at 1 K stage.

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